

# Long-Run Carbon Consumption Risks and Asset Prices

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# This paper

- How environmental policies that aim to reduce carbon emissions affect asset prices and household consumption?
- Do carbon emissions-related risks explain the cross-section of portfolio returns?

# Motivation

- Environmental issues
- **Goods and services consumption** pollutes the environment ;
- Production vs. consumption-based CO2 emissions (GCP).  
▶ Fact
- Despite that, most papers and climate policies focus on the production side.
- Transition to green economy is a long horizon phenomenon (long-run risk) ;
- Need a long-run risk model to assess its effects ;

# Contribution

- Propose a carbon consumption-based risk measure using a novel data ;
- Build a model to study the effects of environmental policies on asset prices, and on household consumption [**Comparative statics**];

# Findings

- 1 Policy : (i) One std dev. decrease in expected carbon consumption (cc) growth, and (ii) One std dev. decrease in the growth of the share of cc :
  - (-) Reduce consumption growth ;
  - (-) Reduce dividend growth, market return ;
  - **Severe during high uncertainty.**
- 2 My long-run risk is more detectable.
  - during periods of high climate change uncertainty.
- 3 My risk factors explain the cross section of size and value sorted portfolios, and industry portfolios.

# Related literature

- **Carbon emissions measurement** : K Song et al. (2019)
  - Prior literature : production approach and consumption approach ;
  - [This paper : consumption approach and novel data.](#)
- **Long-run risk models** : Bansal and Yaron (2004), Bonomo et al. (2011), Constantinides and Ghosh (2011).
  - Prior literature : LRR comes from the aggregate consumption ;
  - [This paper : LRR comes from the carbon consumption.](#)
- **Climate finance** : Bansal et al. (2016), Daniel et al. (2016), Giglio et al. (2021).
  - Prior literature : physical risk ;
  - [This paper : transition risk.](#)

# Overview

- Data construction
- Model
- Empirical evaluation and results

# Data construction

- The central challenge of the climate change is to assess its effect on real economy ;
    - This paper uses consumption and carbon footprint to assess it.
  - This paper uses :
    - National Income and Product Accounts (NIPA)
    - Economic Input-Output Life Cycle Assessment (EIO-LCA)
- ▶ Data
- All consumption data are on annual basis and span the period 1930-2018.



FIGURE – Power generation and supply [▶ Back](#)

<b>Sector</b>		<b>Total t CO2e</b>	<b>CO2 Fossil t CO2e</b>	<b>CO2 Process t CO2e</b>	<b>CH4 t CO2e</b>	<b>N2O t CO2e</b>	<b>HFC/PFCs t CO2e</b>
	<i>Total for all sectors</i>	9370	8880	31.3	346.	56.3	57.5
221100	Power generation and supply	8820	8690	0.000	23.9	54.0	55.9
212100	Coal mining	230	25.9	0.000	204.0	0.000	0.000
211000	Oil and gas extraction	129.0	36.3	23.6	69.0	0.000	0.000
486000	Pipeline transportation	67.1	30.7	0.084	36.3	0.000	0.000
482000	Rail transportation	25.9	25.9	0.000	0.000	0.000	0.000
324110	Petroleum refineries	19.8	19.8	0.000	0.061	0.000	0.000
484000	Truck transportation	9.17	9.17	0.000	0.000	0.000	0.000

# Data construction

- Collect aggregate information on 12 consumption categories using NIPA ;
- Carbon footprints (CFs) on these 12 categories from EIO-LCA. ([▶ Details](#) and [▶ Footprint](#))

# Data construction

- Classify the consumption categories into two groups using the CFs :

- Carbon consumption :

$$CC_t = \sum_{i=1}^5 C_{i,t} \quad (1)$$

- Green consumption :

$$GC_t = \sum_{i=6}^{12} C_{i,t} \quad (2)$$

- Carbon share in the total consumption is :

$$A_{CC,t} = \frac{CC_t}{CC_t + GC_t} = \frac{CC_t}{C_t} \quad (3)$$

- Then  $\Delta cc_t = \Delta \log CC_t$  and  $\Delta \alpha_{cc,t} = \Delta \log A_{CC,t}$
- See [▶ Figure](#)

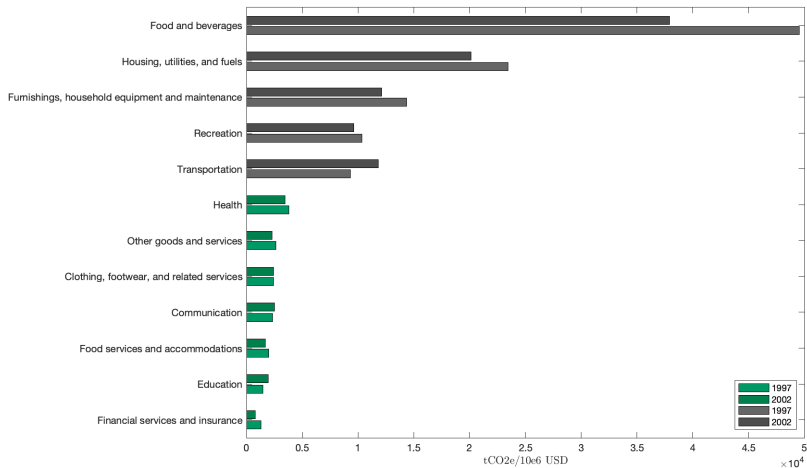
- Consumption-based carbon emissions :

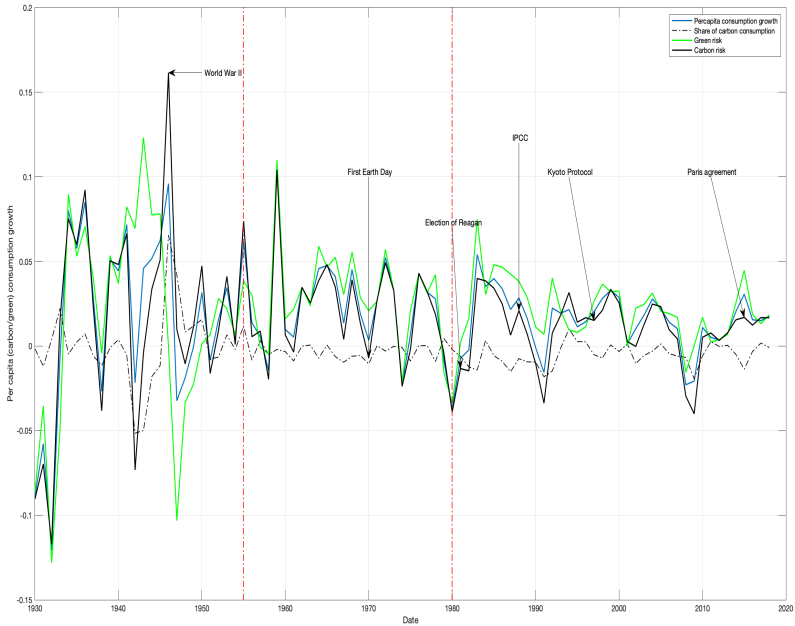
$$CC_t = \sum_{i=1}^{12} CF_i \times C_{i,t} \quad (4)$$

TABLE – Example of footprint at sub-category level. [▶ Back](#)

	1997	2002
<b>Health</b>	-	-
Medical products, appliances, and equipment	-	-
Pharmaceutical and other medical products <sup>9</sup>	-	-
Pharmaceutical products	420	304
Other medical products	379	288
Therapeutic appliances and equipment	1259	1491
Outpatient services	-	-
Physician services <sup>10</sup>	169	157
Paramedical services	-	-
Home health care	197	235
Medical laboratories	558	273
Other professional medical services <sup>11</sup>	na	na
Hospital and nursing home services	-	-
Hospitals <sup>12</sup>	400	366
Nursing homes	443	366
<b>Transportation</b>	-	-
Motor vehicles	-	-
New motor vehicles	382	265
Net purchases of used motor vehicles	622	518
Motor vehicle operation	-	-
Motor vehicle parts and accessories	769	710
Motor vehicle fuels, lubricants, and fluids	3540	2790
Motor vehicle maintenance and repair	423	328
Other motor vehicle services	398	569
Public transportation	-	-
Ground transportation <sup>13</sup>	0	1870
Air transportation	1780	1980
Water transportation	1430	2780

FIGURE – CF by household expenditure category. [▶ Back](#)





## Data construction

- Use the dividend-yield and the dividend growth of the CRSP value-weighted index ;
- Use Fama-French 17 and 30 industries portfolios, and size and book-to-market sorted portfolios.



# Data construction

TABLE – Descriptive statistics : 1930-2018.

▶ Subperiod stats

	$E(\cdot)$	$\sigma(\cdot)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(5)$	$CV$
$\Delta d$	0.0176	0.1223	0.1075	-0.1832	-0.1502	0.0459	6.9325
$\Delta c$	0.0178	0.0343	0.3150	0.0608	-0.1508	0.0098	1.9277
$\Delta\alpha_{cc}$	-0.0030	0.0134	0.4545	0.0574	-0.1233	-0.1730	-4.4179
$\Delta cc$	0.0147	0.0382	0.2717	-0.0223	-0.2057	0.0176	2.5893
$\Delta\alpha_{gc}$	0.0042	0.0215	0.4469	0.0640	-0.0749	-0.2105	5.0928
$\Delta gc$	0.0220	0.0385	0.4647	0.2063	-0.0167	-0.1006	1.7505
$z_m$	3.3878	0.5123	0.9276	0.8524	0.7992	0.7163	0.1512
$r_m$	0.0694	0.1929	0.0077	-0.2202	0.0181	-0.1215	2.7802
$r_f$	0.0025	0.0351	0.6852	0.3059	0.2040	0.2788	14.1068

## Model - Household

- Representative agent economy ;
- Can invest in  $n+1$  assets : one riskless asset ( $i = 0$ ) and  $n$  risky assets ( $i = 1, \dots, n$ ) ;
- Has the following budget constraint :

$$C_t + \sum_{i=1}^{n+1} P_{it} X_{i,t+1} = \sum_{i=1}^{n+1} (P_{it} + D_{it}) X_{it} = W_t \quad (5)$$

- Maximizes an Epstein-Zin utility function :

$$V_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E_t[V_{t+1}]^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (6)$$

$\gamma$  is the CRA and  $\psi = 1 - \frac{1-\gamma}{\theta}$  is the EIS.

- $C = CC$  : carbon consumption +  $GC$  : green consumption :

$$C_t = CC_t + GC_t \quad (7)$$

## Model - LRCCR setting

$$\Delta c_{t+1} = \underbrace{\Delta cc_{t+1}}_{\text{Carbon}} - \underbrace{\Delta \alpha_{cc,t+1}}_{\text{share of cc}} \quad \text{▶ Proof} \quad (8)$$

$$\Delta cc_{t+1} = \nu_{cc} + x_t + \sigma_t \epsilon_{cc,t+1} \quad (9)$$

$$x_{t+1} = \rho_x x_t + \psi_x \sigma_t \epsilon_{x,t+1} \quad (10)$$

$$\sigma_{t+1}^2 = (1 - \nu) \sigma_t^2 + \nu \sigma_t^2 + \sigma_w \epsilon_{\sigma,t+1} \quad (11)$$

$$\Delta \alpha_{cc,t+1} = \nu_\alpha (1 - \rho_\alpha) + \rho_\alpha \Delta \alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi \sigma_t \epsilon_{cc,t+1} \quad (12)$$

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i} \Delta \alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1} \quad (13)$$

$\epsilon_{x,t+1}, \epsilon_{cc,t+1}, \epsilon_{\alpha,t+1}, \epsilon_{i,t+1}$  and  $\epsilon_{\sigma,t+1}$  are i.i.d.

## Model - BY setting

$$\Delta c_{t+1} = \nu_c + x_t + \sigma_t \epsilon_{c,t+1} \quad (14)$$

$$x_{t+1} = \rho_x x_t + \psi_x \sigma_t \epsilon_{x,t+1} \quad (15)$$

$$\sigma_{t+1}^2 = (1 - \nu) \sigma^2 + \nu \sigma_t^2 + \sigma_w \epsilon_{\sigma,t+1} \quad (16)$$

Finally, the dividend of any asset  $i$  growth rate is as follow

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \psi_i \sigma_t \epsilon_{i,t+1} \quad (17)$$

$\epsilon_{x,t+1}, \epsilon_{c,t+1}, \epsilon_{i,t+1}$  and  $\epsilon_{\sigma,t+1}$  are i.i.d.

# Model - Resolution

- For any asset  $i$ , the Euler equation is given by :

$$\mathbb{E}_t(\exp(m_{t+1} + r_{i,t+1})) = 1 \quad (18)$$

where

$$m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} \Delta cc_{t+1} + \frac{\theta}{\psi} \Delta \alpha_{cc,t+1} + (\theta - 1) r_{c,t+1} \quad (19)$$

- Innovation in the SDF :

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_\alpha \underbrace{\sigma_\alpha \epsilon_{\alpha,t+1}}_{\text{g.r}} - \lambda_{cc} \underbrace{\sigma_t \epsilon_{cc,t+1}}_{\text{c.r}} - \lambda_x \underbrace{\sigma_t \epsilon_{x,t+1}}_{\text{l.r.r}} - \lambda_w \underbrace{\sigma_w \epsilon_{\sigma,t+1}}_{\text{v.r}}$$

- Four risks : green risk, carbon risk, long-run risk and the volatility risk ;
- Expected equity premium of any asset  $i$  :

$$\mathbb{E}_t(r_{i,t+1} - r_{f,t}) = \lambda_x \beta_{i,x} \sigma_t^2 + \lambda_w \beta_{i,w} \sigma_w^2 + \lambda_\alpha \beta_{i,\alpha} \sigma_\alpha^2 + \lambda_{cc} \beta_{i,cc} \sigma_t^2$$

# Empirical evaluation strategy

- Period where no effect on firm-level nor industry-level returns.
- Identify key period where climate change matters become a public debate ;
  - Goal : as of which date or period asset prices start reflecting carbon-consumption risk.
- We split our sample into three sub-periods for that reason :
  - 1930-1955 (Great economic uncertainty period) ;
  - 1956-1980 (Oil crisis period) ;
  - 1981-2018 (More climate action period).

# Empirical evaluation strategy

- 1 Calibrate  $\Theta = [\rho_x \ \psi_x \ \psi_i \ \nu_{cc} \ \nu \ \nu_i \ \sigma_w \ \sigma \ \phi_i \ \delta \ \gamma \ \psi \ \nu_\alpha \ \rho_\alpha \ \sigma_\alpha \ \pi \ \phi_{\alpha,i}]$  to match key data moments
- 2 Policy simulation
- 3 Asset pricing implications :
  - Testing the equity premium, volatility and risk-free rate puzzles :
    - $r_{f,t} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t}$
    - $\mathbb{E}_t r_{i,t+1} = B_0 + B_1x_t + B_2\sigma_t^2 + B_3\Delta\alpha_{cc,t}$
  - Pricing portfolios (Cross-section of assets) :
    - $r_{i,t} = c_i + \beta_{ff3,i}FF3_t + \beta_{sh,i}\Delta\alpha_{cc,t} + \beta_{cc,i}\Delta cc_t + \epsilon_{i,t} \quad i = 1 \dots n$
    - $r_{i,t} = c_i + \beta_{ff3,i}FF3_t + \beta_{sh,i}MP_{\alpha,t} + \beta_{cc,i}MP_{cc,t} + \epsilon_{i,t}$
    - Risk premium estimates of each factor.

TABLE – Calibrated parameters

	1930-2018		1930-1955		1956-1980		1981-2018	
	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR
$\rho_x$	0.932	0.978	0.937	0.979	0.920	0.900	0.976	0.900
$\psi_x$	0.259	0.150	0.278	0.119	0.010	0.204	0.206	0.514
$\psi_d$	4.540	4.340	4.789	4.488	13.361	0.000	10.122	4.288
$\nu_x$	9E-04	1E-03	1E-04	1E-03	-6E-05	2E-03	-2E-04	9E-04
$\nu$	0.999	0.979	0.573	0.985	0.577	0.691	0.988	0.995
$\nu_d$	0.001	-0.011	0.000	-0.025	-0.002	0.001	0.005	0.003
$\sigma_w$	5E-07	2E-08	1E-04	2E-07	7E-08	1E-05	4E-06	4E-06
$\sigma$	8E-03	3E-03	5E-04	4E-03	1E-03	1E-03	7E-04	9E-03
$\phi$	2.294	3.378	2.354	3.734	321.850	10.056	0.792	1.019
$\delta$	0.956	0.998	0.999	0.999	0.998	0.998	0.998	0.997
$\gamma$	7.074	12.290	9.878	10.084	15.940	23.016	6.063	8.732
$\psi$	1.379	1.487	3.018	1.495	1.574	1.235	1.503	1.486
$\bar{z}$	3.088	6.164	6.054	6.602	6.201	6.285	5.720	5.060
$\bar{z}_m$	5.344	3.981	5.153	3.522	4.754	5.696	12.820	5.548
$\nu_a$		-3E-04		4E-05		-3E-04		-4E-04
$\rho_a$		0.455		0.480		-0.281		0.360
$\sigma_a$		0.006		0.006		0.014		0.004
$\pi$		1.344		0.897		3.328		0.626
$\phi_a$		0.590		0.877		-0.294		1.305



# Takeaway

- $\Psi_x$  tells us how detectable the LRR is.  $\Psi_{x,LRCCR} > \Psi_{x,BY}$  near climate change events periods.
- LRR in volatility and in expected cc growth :
  - $\nu$  smaller and close to one ;
  - $\rho_x$  smaller and close to one.
- $\gamma_{LRCCR} > \gamma_{BY}$  and is reasonable ;
  - Agents fear more carbon risk than consumption risk.

TABLE – Model-implied moments. [▶ More](#)

	1956-1980			1981-2018		
	Data	BY2004	LRCCR	Data	BY2004	LRCCR
$\sigma(z_m)$	0.182	0.125	<b>0.175</b>	0.415	0.078	0.204
$EP$	0.041	0.093	<b>0.068</b>	0.072	<b>0.066</b>	0.118
$E(R_f)$	0.003	-0.002	<b>0.003</b>	0.011	<b>0.010</b>	<b>0.015</b>
$\sigma(r_{m,a})$	0.178	0.134	<b>0.197</b>	0.162	0.128	<b>0.183</b>
$\sigma(r_{f,a})$	0.003	0.001	0.019	0.011	0.002	0.006
$\rho(z_m)$	0.656	0.345	0.282	0.890	0.846	0.809

- Can the LRR predict the risk premium?
  - **Yes** during period of high uncertainty. A long-run carbon consumption risks model finds a time-varying risk premium.
  - Much better than the usual LRR model.
  - Otherwise, approximately constant risk-premium.

# Policy simulation

TABLE – Comparative statics

Policy	$\Delta c$	$r_m - r_f$	$\Delta cc$
1930-2018	<b>-26.79</b>	-3.36	-32.72
1930-1955	-9.48	-0.80	-9.48
1956-1980	-0.56	-0.17	-0.69
1981-2018	<b>-23.14</b>	-5.96	-34.83

# Policy simulation : Impulse response

TABLE – Comparative statics : IRF 1981-2018.

▶ Graph

▶ 1930-2018

	$\epsilon_x$ shock	$\epsilon_{cc}$ shock	$\epsilon_\alpha$ shock
$\Delta cc$	10.71	1.85	0.00
$\Delta \alpha_{cc}$	0.00	2.54	0.73
$\Delta d$	11.52	2.36	0.68
$z_m$	63.35	4.62	1.34
$r_m$	6.51	-0.60	-0.17
$r_f$	7.04	-0.60	-0.17

# Takeaway

- The effects of  $\epsilon_x$  shock is more severe ;
- The impacts is more severe over 1981-2018 period.
  - Due to high uncertainty ;
  - Combine with the persistent property of  $x$ .

## Cross-sectional implications

- In this paper, I decompose the consumption growth risk  $\Delta c_t$  into two :
  - Carbon consumption (cc) growth risk ( $\Delta cc_t$ );
  - Share of carbon consumption growth risk ( $\Delta \alpha_t$ ).
- Add Fama and French (1993) three factors : *mkt*, *hml*, and *smb*
- Compute the risk premium estimates using two-pass CSR :
  - Raw factors [non tradable],
  - Mimicking portfolios of my factors [tradable].
- Assess the contribution of my factors.

# Cross-sectional implications

TABLE – Industry portfolios. [Annual]

	mkt	hml	smb	$\Delta_{cc}$	$\Delta_{\alpha_{cc}}$	$R^2$
$\hat{\gamma}(\%)$	-2.59			-0.84	-4.1E-04	
t ratio	-1.08			-1.69	-0.03	0.059
$\hat{\gamma}(\%)$	-2.47	-4.04	-0.59			
t ratio	-1.02	-2.30	-0.38			0.095
$\hat{\gamma}(\%)$	-2.45	-4.23	-0.53	-0.89		
t ratio	-1.02	-2.40	-0.34	-1.79		0.122
$\hat{\gamma}(\%)$	-2.54	-4.04	-0.59		-3.2E-03	
t ratio	-1.05	-2.30	-0.37		-0.26	0.097
$\hat{\gamma}(\%)$	-2.39	-4.25	-0.53	-0.89	-1.2E-03	
t ratio	-0.99	-2.40	-0.34	-1.79	-0.09	0.120



# Cross-sectional implications

TABLE – FF25P. [Annual]

	mkt	hml	smb	$\Delta cc$	$\Delta \alpha_{cc}$	$R^2$
$\hat{\gamma}(\%)$	-9.44			-1.10	-0.07	
t ratio	-3.17			-2.07	-4.07	0.191
$\hat{\gamma}(\%)$	-12.24	4.06	2.48			
t ratio	-4.01	2.58	1.71			0.250
$\hat{\gamma}(\%)$	-11.47	4.07	2.45	-0.85		
t ratio	-3.66	2.59	1.68	-1.61		0.259
$\hat{\gamma}(\%)$	-12.22	4.20	1.99		-0.06	
t ratio	-4.00	2.67	1.36		-3.63	0.350
$\hat{\gamma}(\%)$	-12.59	4.20	1.98	-1.06	-0.06	
t ratio	-4.00	2.67	1.36	-2.00	-3.66	0.351

# Cross-sectional implications. Mimicking Portfolios

TABLE – FF25P. [Monthly]

	mkt	hml	smb	$MP_{cc}$	$MP_{\alpha}$	$R^2$
$\hat{\gamma}(\%)$	0.029			-0.035	<b>-0.001</b>	
t ratio	0.11			-1.55	-2.69	0.1152
$\hat{\gamma}(\%)$	-0.711	0.336	0.238			
t ratio	-2.39	3.13	2.44			0.3491
$\hat{\gamma}(\%)$	-0.876	0.336	0.239	<b>-0.050</b>		
t ratio	-2.89	3.13	2.45	-2.21		0.4594
$\hat{\gamma}(\%)$	-0.572	0.337	0.237		<b>-0.001</b>	
t ratio	-1.89	3.14	2.43		-2.43	0.4448
$\hat{\gamma}(\%)$	-0.738	0.337	0.238	<b>-0.048</b>	<b>-0.001</b>	
t ratio	-2.36	3.14	2.44	-2.11	-2.36	0.5040

# Takeaway

- My risk factors perform better at explaining FF25P.
  - Carbon factors are negatively priced
  - Green factors are positively priced.

# Conclusion

- Provide a model to analyze how environmental policies that aim to reduce carbon emissions affect asset prices and household consumption.
- Provide new risk factors to explain the cross-section of portfolios.

# Research agendas

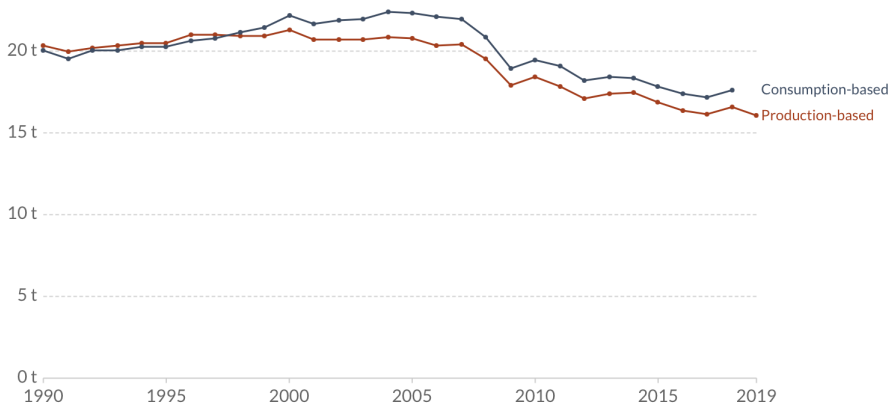
- Asset Pricing using Nonlinear Principal Components, with Rene Garcia and Caio Almeida.
  - Use truly independent nonlinear factors to predict future returns ;
  - Fama-French 25 ME/BM- sorted portfolios and fifty anomaly portfolios ;
  - SDF estimated using a mixture of nonlinear and linear factors outperform the one using solely linear factors or raw characteristic returns ;
  - Our hybrid model requires less risk factors to achieve the highest out-of-sample performance compared to a model using only linear factors.

# Research agendas

- Insurance Asset Pricing
  - Household asset pricing ;
  - Intermediary asset pricing **since 2013** ;
    - Lens of frictions in financial intermediation.
  - Due to climate change, Insurers will be more affected.
    - → **Insurance Asset Pricing**
- Hedging Physical and Transition Climate Change Risks
  - Build transition risk using textual analysis ;
    - **$transrisk = f(news)$**
  - Build physical risk using data on natural disasters ;
    - **$physrisk = f(hurricanes, temp, sealevelrise, other)$**
  - Build hedging portfolio : mimicking portfolio approach.
    - **$transrisk_t = \alpha_{tr} + w'_{tr}er_t + \epsilon_{tr,t}$**
    - **$physrisk_t = \alpha + w'_{pr}er_t + \epsilon_{pr,t}$**

THANK YOU!!!

FIGURE – Carbon emissions in USA



Source: Our World in Data based on the Global Carbon Project and UN Population  
OurWorldInData.org/co2-and-other-greenhouse-gas-emissions • CC BY

▶ Back



Let us consider  $I$  categories of consumption among which  $J$  carbon consumption categories and  $I - J$  green consumption categories.

$$C_t = \sum_{i=1}^I C_{i,t} \quad (20)$$

$$C_t = \sum_{i=1}^J C_{i,t} + \sum_{i=J+1}^I C_{i,t} \quad (21)$$

$$C_t = CC_t + GC_t \quad (22)$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \quad (23)$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log \frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log \frac{CC_t + GC_t}{CC_t} \quad (24)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left( \log \frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log \frac{CC_t}{CC_t + GC_t} \right) \quad (25)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta \alpha_{CC,t+1} \quad (26)$$

where  $\Delta c_{t+1}$ ,  $\Delta cc_{t+1}$  and  $\Delta \alpha_{CC,t+1}$  are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively. ▶ LRCCR

For this decomposition, we weight each consumption category by its impact on the environment measured by their carbon footprints.

$$C_t = \sum_{i=1}^I CF_i \times C_{i,t} \quad (27)$$

$$C_t = \sum_{i=1}^J CF_i \times C_{i,t} + \sum_{i=J+1}^I CF_j \times C_{i,t} \quad (28)$$

$$C_t = CC_t + GC_t \quad (29)$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \quad (30)$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log \frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log \frac{CC_t + GC_t}{CC_t} \quad (31)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left( \log \frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log \frac{CC_t}{CC_t + GC_t} \right) \quad (32)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta \alpha_{CC,t+1} \quad (33)$$

where  $\Delta c_{t+1}$ ,  $\Delta cc_{t+1}$  and  $\Delta \alpha_{CC,t+1}$  are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively. [▶ LRCCR](#)

TABLE – Descriptive statistics : 1930-1955. ▶ Model

	$E(\cdot)$	$\sigma(\cdot)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(5)$	$CV$
$\Delta d$	0.0038	0.1914	0.1244	-0.2814	-0.1927	0.0954	50.6037
$\Delta c$	0.0169	0.0540	0.3633	0.1214	-0.1689	0.0208	3.1977
$\Delta\alpha_{cc}$	<b>0.0005</b>	0.0231	0.4800	0.0383	-0.1503	-0.2569	50.1958
$\Delta cc$	0.0173	0.0611	0.2899	-0.0096	-0.2380	0.0343	3.5250
$\Delta\alpha_{gc}$	<b>-0.0009</b>	0.0378	0.4771	0.0408	-0.0949	-0.2775	-42.0192
$\Delta gc$	0.0160	0.0621	0.5495	0.2875	-0.0174	-0.1701	3.8876
$z_m$	2.8535	0.2265	0.4185	-0.0828	-0.2764	-0.2802	0.0794
$r_m$	0.0732	0.2468	0.0904	-0.2068	-0.0779	-0.0288	3.3692
$r_f$	-0.0103	0.0558	0.6365	0.1463	0.0169	0.2074	-5.4163

TABLE – Descriptive statistics : 1956-1980. ▶ Model

	$E(\cdot)$	$\sigma(\cdot)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(5)$	$CV$
$\Delta d$	0.0074	0.0523	0.2667	0.0359	0.0091	0.0053	7.0994
$\Delta c$	0.0222	0.0290	-0.0204	-0.2613	-0.0383	0.1899	1.3081
$\Delta\alpha_{cc}$	<b>-0.0035</b>	0.0043	-0.2815	-0.0210	0.3359	0.0456	-1.2429
$\Delta cc$	0.0187	0.0295	-0.0577	-0.2755	-0.0490	0.1629	1.5790
$\Delta\alpha_{gc}$	<b>0.0061</b>	0.0076	-0.2780	-0.0079	0.3276	0.0584	1.2326
$\Delta gc$	0.0283	0.0296	0.0243	-0.2123	0.0213	0.2276	1.0473
$z_m$	3.2918	0.1822	0.6555	0.3480	0.2547	0.1185	0.0553
$r_m$	0.0445	0.1779	-0.0762	-0.3736	0.1056	0.0922	4.0023
$r_f$	0.0030	0.0152	0.5917	0.4075	0.4660	0.2736	5.0496

TABLE – Descriptive statistics : 1981-2018. ▶ Model

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(5)$	$CV$
$\Delta d$	0.0339	0.0926	-0.0365	-0.0047	-0.0714	-0.0885	2.7331
$\Delta c$	0.0155	0.0162	0.4603	0.0530	-0.0526	-0.0747	1.0438
$\Delta\alpha_{cc}$	<b>-0.0052</b>	0.0066	0.3599	-0.0832	-0.2421	0.1352	-1.2812
$\Delta cc$	0.0104	0.0185	0.5090	0.0042	-0.1916	-0.1158	1.7807
$\Delta\alpha_{gc}$	<b>0.0065</b>	0.0083	0.3914	-0.0141	-0.1654	0.1809	1.2843
$\Delta gc$	0.0220	0.0172	0.3744	0.1211	0.1178	0.0877	0.7844
$z_m$	3.8166	0.4151	0.8895	0.7548	0.6669	0.4293	0.1088
$r_m$	0.0831	0.1618	-0.0695	-0.1204	0.0703	-0.4177	1.9464
$r_f$	0.0109	0.0221	0.7975	0.6500	0.5337	0.3189	2.0303

TABLE – Model-implied moments. [▶ Back](#)

	$\sigma(z_m)$	$EP$	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$	
1930-1955							
BY2004	Data	0.227	0.084	-0.010	0.247	0.010	0.418
	Mean	0.238	0.109	0.007	0.264	0.159	0.354
	5%	0.166	-0.019	-0.051	0.193	0.122	0.019
	50%	0.234	0.105	0.006	0.261	0.158	0.368
	95%	0.325	0.251	0.065	0.346	0.198	0.637
LRCCR	Mean	0.150	0.040	0.010	0.116	0.019	0.608
	5%	0.098	0.000	-0.005	0.089	0.014	0.291
	50%	0.145	0.039	0.010	0.115	0.018	0.635
	95%	0.216	0.081	0.024	0.144	0.025	0.834
1956-1980							
BY2004	Data	0.182	0.041	0.003	0.178	0.003	0.656
	Mean	0.125	0.093	-0.002	0.134	0.001	0.345
	5%	0.093	0.045	-0.002	0.102	0.001	0.019
	50%	0.124	0.093	-0.002	0.133	0.001	0.360
	95%	0.162	0.143	-0.001	0.167	0.002	0.620
LRCCR	Mean	0.175	0.068	0.003	0.197	0.019	0.282
	5%	0.119	0.003	-0.005	0.138	0.014	-0.041
	50%	0.172	0.066	0.003	0.194	0.019	0.293
	95%	0.240	0.141	0.010	0.263	0.024	0.569

FIGURE – 1930-2018 [▶ Back](#)

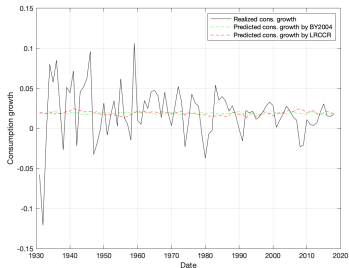
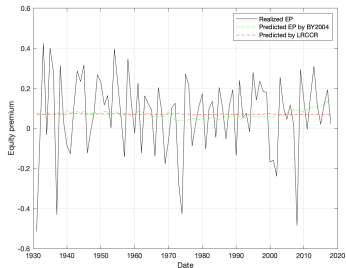


FIGURE – 1981-2018 [▶ Back](#)

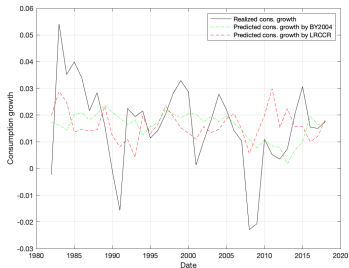
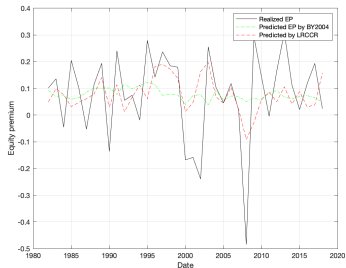




FIGURE – Policy : 1981-2018 [▶ Back](#)

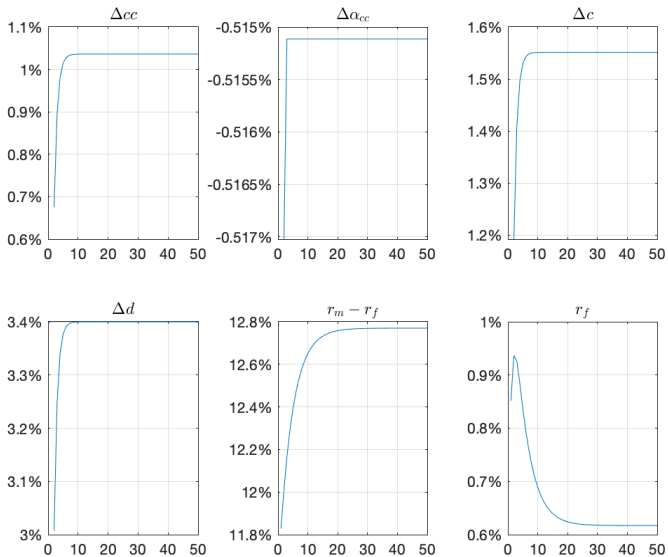


FIGURE – IRF : 1981-2018 [▶ Back](#)

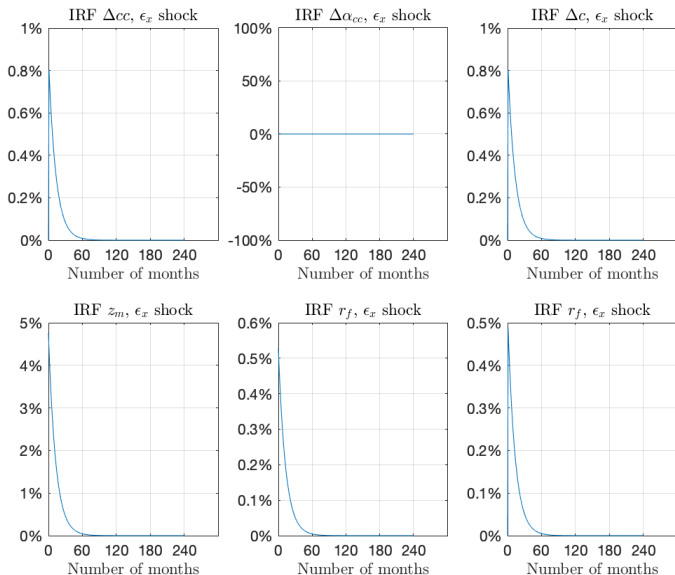


FIGURE – IRF : 1930-2018 [▶ Back](#)

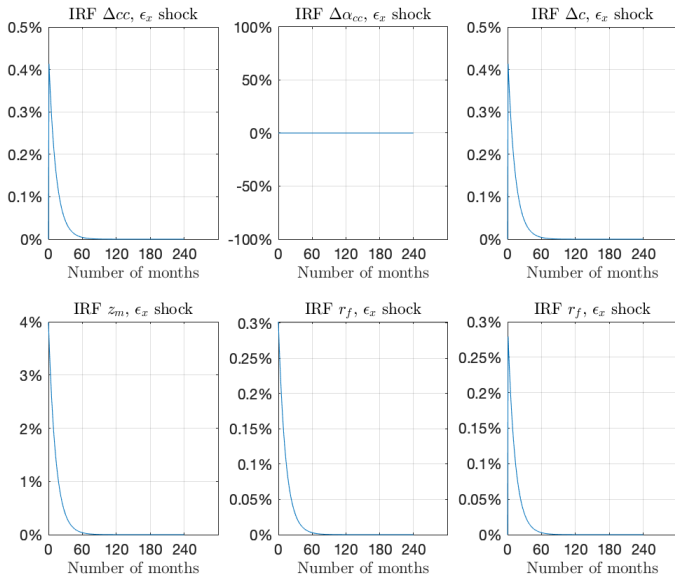


FIGURE – IRF : 1930-1955 [▶ Back](#)

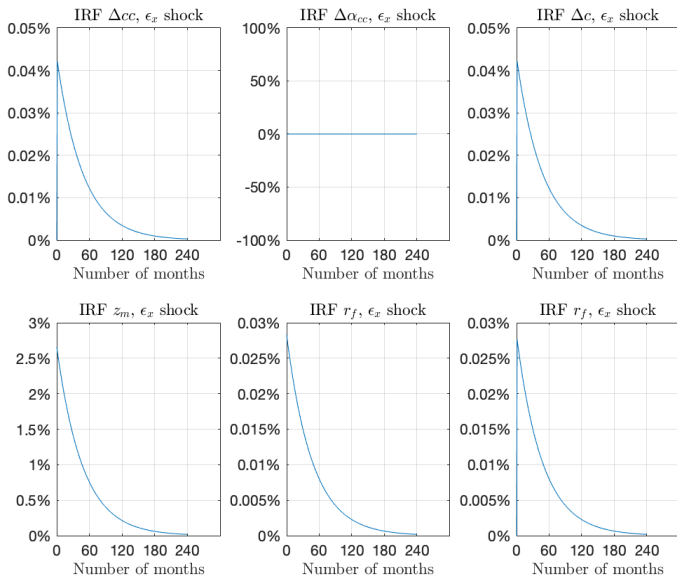


FIGURE – IRF : 1956-1980 [▶ Back](#)

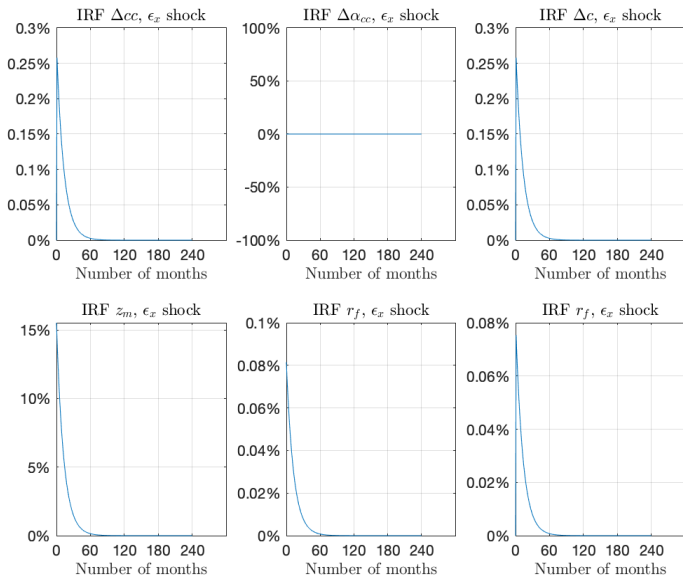


TABLE – Comparative statics : IRF 1930-2018 [▶ Back](#)

	$\epsilon_x$ shock	$\epsilon_{cc}$ shock	$\epsilon_\alpha$ shock
$\Delta cc$	5.51	1.29	0.00
$\Delta \alpha_{cc}$	0.00	1.25	1.49
$\Delta d$	7.92	1.02	1.21
$z_m$	53.15	2.62	3.12
$r_m$	3.72	-0.41	-0.49
$r_f$	4.02	-0.41	-0.49

# Contribution to the CS $R^2$

TABLE – Testing the difference in terms of CS  $R^2$

Industries	$\Delta_{cc}$	$\Delta_{\alpha_{cc}}$	$\Delta_{cc} + \Delta_{\alpha_{cc}}$	$\Delta_{gc}$	$\Delta_{\alpha_{gc}}$	$\Delta_{gc} + \Delta_{\alpha_{gc}}$	$\Delta_c$
$R_{ff3}^2 - R_{new}^2$	-0.027	-0.002	-0.028	-0.004	-0.006	-0.027	-0.075
$p_{cs}$	0.146	0.753	0.391	0.638	0.606	0.446	0.011
$p_{ms}$	0.248	0.787	0.544	0.692	0.648	0.571	0.059
$p_{wald,cs}$	0.146	0.753	0.241	0.638	0.606	0.305	0.011
$p_{wald,ms}$	0.248	0.787	0.476	0.692	0.648	0.505	0.059
<hr/>							
FF25P							
$R_{ff3}^2 - R_{new}^2$	-0.009	-0.100	-0.101	-0.046	-0.092	-0.093	-0.004
$p_{cs}$	0.294	0.022	0.044	0.076	0.048	0.082	0.510
$p_{ms}$	0.530	0.074	0.162	0.162	0.096	0.193	0.689
$p_{wald,cs}$	0.294	0.022	0.081	0.076	0.048	0.143	0.510
$p_{wald,ms}$	0.530	0.074	0.221	0.162	0.096	0.267	0.689

# Takeaway

- Under correctly specify model assumption :
  - Three alternative specifications outperform FF3 model at 5% level
  - Two outperform at 10% level.
- Under misspecification assumption :
- Two of my alternative models outperform at 10% and none at 5%.



FIGURE – FF3 versus alternative specifications

